Fermi National Accelerator Laboratory

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Copper Coating the TEVATRON Beam Pipe

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Operated by Universities Research Association Inc. under contract with the United States Department of Energy

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I. INTRODUCTION

There has been several plans to upgrade the Tevatron. One of which involves filling nearly all the rf buckets with proton bunches to collide with bunches from a new similarly built Tevatron ring.[1] With the introduction of so many bunches, one needs to worry the collective coupled motions of the bunches and also the parasitic power loss to the resistivity of the vacuum chamber walls. In this paper, we address the possibility of copper coating the interior of the vacuum chamber walls. Since the conductivity of copper at cryogenic temperature is ~ 1000 higher than that of stainless steel, collective coupled bunch motions as well as parasitic heating should be greatly reduced. These effects are discussed in Sections II and III.

However, the coating can bring about mechanical problems to the beam pipe. During a quench, the Lorentz force generated by the eddy currents in the highly conductive copper layer can lead to deformation of the beam pipe. The partially vaporized helium outside, if not allowed to escape fast enough, can easily push the beam pipe towards stress failure. Also if the copper coating is not even, this Lorentz force generated during a quench will produce a torque, which if strong enough can cause failure to the position-fixing keys attached to the beam pipe. These problems are studied in Sections IV and V.

For the Tevatron ring, we take the mean radius as $R = 1.0$ km, the beam pipe as approximately cylindrical with an internal radius $b = 3.49$ cm and a wall thickness of $t = 1.0$ mm. At cryogenic temperatures, the conductivity of bulk copper is taken as $\sigma_c = 2.3 \times 10^9$ $(\Omega\text{-m})^{-1}$ corresponding to a residual resistance ratio (RRR) of 200. The conductivity of stainless steel is taken as $\sigma_s = 2.0 \times 10^6$ $(\Omega\text{-m})^{-1}$.

II. TRANSVERSE COUPLED-BUNCH GROWTHS

The rf harmonic of the Tevatron is $h = 1113$. But only $M = 996$ buckets will be filled with bunches each containing $N = 2.0 \times 10^{10}$ protons.[1] For simplicity let us assume that all the buckets are filled, or $M = h$. There will be M modes of transverse coupled-bunch motion. Here, internal bunch motions have not been considered. The growth rate of the μth mode driven by the transverse coupling impedance $Z_\perp$ is[2]

$$\frac{1}{\tau_\mu} = -\frac{MI_bc}{4\pi\nu_\beta E/e} \sum_{k=-\infty}^{\infty} \mathcal{R}e\, Z_\perp[(kM-\mu+\nu_\beta)\omega_0] \,, \tag{2.1}$$

where E is the energy per particle in the bunch, I_b is the average current per bunch, c is the velocity of light, ν_β is the betatron tune, and $\omega_0/2\pi$ is the mean revolution frequency. Since $\mathcal{R}e\, Z_\perp(\omega)$ is odd in ω, broad-band impedance with a width larger than $\sim \nu_\beta\omega_0$ will not contribute. The wall resistivity behaves like a resonance at low frequencies because it goes to infinity when $\omega = 0$. As a result, its contribution to Eq. (2.1) is significant. At harmonic n, the skin depth into the vacuum chamber wall is given by

$$\delta = \frac{1}{\sqrt{n}} \left(\frac{2R}{\beta Z_0 \sigma} \right)^{1/2} , \tag{2.2}$$

where βc is the average longitudinal velocity of the beam particle, $Z_0 = 377$ ohms is the free-space impedance, and σ is the electric conductivity of the chamber wall. We get for stainless steel $\delta = 1.63 n^{-1/2}$ mm. Therefore for frequency less than a harmonic, the return image currents fill the whole wall thickness of $t = 1$ mm. The transverse resistive wall impedance of the whole ring is given by

$$Z_\perp(\omega) = \frac{2Rc}{\sigma_s t \omega b^3} . \tag{2.3}$$

The ring has a designed betatron tune of $\nu_\beta = 20.6$. Therefore the coupled-bunch mode that has the fastest growth is $\mu = 21$, corresponding to the spectral line with $k = 0$ or at $\omega = -0.4\omega_0$ where $Z_\perp = -58.8$ MΩ/m for a stainless steel pipe wall. The next adjacent spectral lines corresponding to $k = \pm 1$ are $M = 1113$ units of ω_0 away and give negligible contributions (Fig. 1). The designed average bunch current is $I_b = eN\beta c / 2\pi R \approx 0.153$ mA. The worst growth rate occurs at injection energy of $E = 150$ GeV and is given by

$$\frac{1}{\tau_{21}} = 77.4 \text{ sec}^{-1} , \tag{2.4}$$

or a growth time of $\tau_{21} = 12.9$ ms.

For the Superconducting Super Collider (SSC), any transverse coupled-bunch growth times $\tau > 8.5$ ms can be stabilized by the designed transverse damper. Using this as a criterion, the present Tevatron stainless steel beam pipe should be controllable against transverse coupled-bunch instabilities and no copper coating is required. However, our result depends very critically on the residual betatron tune. If $\nu_\beta = 20.9$ say, τ_{21} becomes $12.9 \times (21 - 20.9)/(21 - 20.6) = 3.2$ ms. Also if the number of protons per bunch is increased to 1.0×10^{11}, the growth time will be decreased 5 times to only $\tau_{21} = 2.6$ ms. Then, the instability will become an important issue. On the other hand, with a copper layer of thickness $t_c = 1$ mil (25.4 micron) coated inside the beam pipe everywhere, the factor $\sigma_s t$ in Eq. (2.3) should be replaced by $\sigma_s t + \sigma_c t_c$. Thus, the transverse wall impedance is reduced by $(\sigma_s t + \sigma_c t_c)/\sigma_s t = 30.2$ times, and the growth time is increased by the same ratio.

III. PARASITIC HEATING

The power lost to wall resistivity can be obtained by integrating the longitudinal coupled wall impedance over the bunch power spectrum to get[3]

$$P = \Gamma(3/4)M(I_bR)^2\left(\frac{Z_0}{2\sigma}\right)^{1/2}\left(\frac{1}{b\sigma_z^{3/2}}\right) , \tag{3.1}$$

where $\Gamma(3/4)=1.23$ is the gamma function, $Z_0=377\ \Omega$, σ_z is the rms bunch length, and σ is the conductivity of the pipe wall. Here we have neglected all contributions due to higher multipoles and assumed that the wall currents flow in a skin depth of the pipe wall only.

The Tevatron bunch has a typical rms length of $\sigma_z = 20$ cm. With $M = 996$ bunches, the parasitic heating amounts to $P=2247$ watts for stainless steel pipe wall. The load for each Tevatron refrigerator is listed in Table I. There are 24 refrigerators with a total load of 8.28 kW DC and 21.8 kW AC. Thus, the parasitic heating of the pipe wall contributes to only $\sim 10\%$ of present load and is not significant at all. However, with the number per bunch increases by 5 times to $N = 1.0 \times 10^{11}$, the heating power will be increases by 25 times to $P = 56.2$ kW. Then, the heat generated by the wall resistivity will become a dominant contribution to the load of the refrigerators.

	DC (watt)	AC (watt)
34 dipoles	238.0	238.0
34 dipoles AC		442.0
11 quads	77.0	77.0
11 quads		121.0
1 pair 5000 A lead	10.0	10.0
Set end boxes	20.0	20.0
Total	345.0	908.0

Table I: DC and AC loads of one Tevatron refrigerator.

If the pipe wall is coated with a copper layer of thickness t_c everywhere, the power lost to the wall is still given by Eq. (3.1) with the substitution

$$\frac{I_b^2}{\sigma^{1/2}} \to \frac{I_c^2}{\sigma_c^{1/2}} + \frac{I_s^2}{\sigma_s^{1/2}} , \tag{3.2}$$

where I_c and I_s are the image currents flowing in the copper layer and the stainless steel pipe respectively. The copper layer and the stainless steel pipe can be viewed as two resistances in parallel. Thus,

$$\frac{I_s}{I_{cu}} = \frac{\sigma_s t}{\sigma_c t_c} , \tag{3.3}$$

or

$$\frac{I_c}{I_b} = \frac{1}{1+\sigma_s t/\sigma_c t_c} \qquad \text{and} \qquad \frac{I_s}{I_b} = \frac{\sigma_s t/\sigma_c t_c}{1+\sigma_s t/\sigma_c t_c} . \tag{3.4}$$

Therefore,

$$\frac{I_c^2}{\sigma_c^{1/2}} + \frac{I_s^2}{\sigma_s^{1/2}} = \frac{I_b^2}{\sigma_c^{1/2}} \left\{ \frac{1}{(1+\sigma_s t/\sigma_c t_c)^2} \left[1 + \left(\frac{\sigma_c}{\sigma_s}\right)^{1/2} \left(\frac{\sigma_c t_c}{\sigma_s t}\right)^2 \right] \right\} . \tag{3.5}$$

With $t_c = 1$ mil, $I_s/I_c = \sigma_s t/\sigma_c t_c = 0.0342$ and the factor in the curly brackets is $r = 0.972$, showing that most of the power loss comes from the copper layer. The parasitic heating will be reduced by a factor $\sqrt{\sigma_c/\sigma_s}/r \sim 35$ times. Therefore, for a copper layer that is not too thin, the reduction is almost independent of the layer thickness.

To safeguard the beam pipe against stress failure, however, it is found below that only a copper layer of thickness 0.1 mil is allowed. Then, $I_s/I_c = \sigma_s t/\sigma_c t_c = 0.342$ and the factor in the curly brackets of Eq. (3.5) takes the value $r = 2.76$. Thus, the parasitic heating is reduced by only 12.3 times. The drop in the reduction factor is due to the fact that only about 75% of the image current flows in the copper layer.

IV. STRESS FAILURE DURING A QUENCH

When a quench occurs, the helium outside the beam pipe will be partially vaporized and the external helium pressure will rise. If the beam pipe of average radius $\bar{b}$ and thickness t is perfectly cylindrical, it will be buckled when the radially symmetric helium pressure p reaches the critical buckling pressure

$$p_c = \frac{Yt^3}{4(1-\mu^2)\bar{b}^3} , \tag{4.1}$$

where Y is the modulus of tensile elasticity and μ the Poisson's ratio for lateral contraction. The buckled pipe can reach the elastic limit easily when the helium pressure exceeds p_c slightly. Note that p_c is very sensitive to the radius to thickness ratio $m = \bar{b}/t$.

For the analysis in this section and the next, we shall approximate the proportional limit, below which Hooke's law holds, by the yield limit. For many materials, the yield limit comes just slightly beyond the proportional limit.

However, besides the radially symmetric exterior helium pressure, there is another horizontal force if the beam pipe is copper coated. These forces are illustrated in Fig. 2. During a quench, the sudden drop in vertical magnetic dipole flux density induces eddy currents in the pipe wall which flows mainly in the highly conductive copper layer. Thus, a horizontal outward Lorentz pressure

$$P_L(\varphi) = P_m \sin\varphi \ , \tag{4.2}$$

is generated, where φ is the polar angle. This pressure has a maximum at the equator, $\varphi = \pi/2$, given by

$$P_m = (\sigma_c t_c + \sigma_s t) b |B\dot{B}| \ . \tag{4.3}$$

Here, the conductivity of copper σ_c, the dipole magnetic flux density B, and its time derivative $\dot{B}$ are all time dependent during the quench, but the maximum is implied in Eq. (4.3). Note that this Lorentz pressure is directly proportional to the thickness of the copper layer if the layer thickness is not too small. Equation (4.3) is in mks units. If B is in tesla, $\dot{B}$ in tesla/sec, b and t_c in m, and σ_c in $(\Omega\text{-m})^{-1}$, then the pressure P_m will be in newton/m^2. To convert to psi or atmospheres, the factors 1 newton/m$^2 = 1.45 \times 10^{-4}$ psi and 1 atm $= 14.68$ psi should be used.

With such a Lorentz pressure, the beam pipe will be flattened. Then the helium pressure required to further deform the pipe to the elastic limit will be less than the critical buckling pressure p_c. For a given equatorial Lorentz pressure P_m, the yield helium pressure p is given by[4]

$$\sigma_{yp} = \begin{cases} p\dfrac{\bar{b}}{t} + \dfrac{3P_m}{2}\dfrac{\bar{b}^2}{t^2}\dfrac{p_c}{p_c - p} & \text{when } P_m < 2p \\[2ex] (P_m - p)\dfrac{\bar{b}}{t} + \dfrac{3P_m}{2}\dfrac{\bar{b}^2}{t^2}\dfrac{p_c}{p_c - p} & \text{when } P_m > 2p \ , \end{cases} \tag{4.4}$$

where σ_{yp} is the tensile yield stress of the pipe material. For the first relation of Eq. (4.4), the pipe yields at the equatorial points $\varphi = \pm\pi/2$, and for the second relation, the pipe yields at the polar points $\varphi = 0$ and π.

Taking for stainless steel at 20 K, the modulus of tensile elasticity $Y = 30 \times 10^6$ psi, the Poisson's ratio $\mu = 0.278$, and the yield-point stress $\sigma_{yp} = 115 \pm 25$ kpsi, the limiting helium and Lorentz pressures are computed and are plotted in Fig. 3 with the range of uncertainty indicated by dotted curves. Here, we take the average pipe radius as $\bar{b}=3.49+0.05=3.54$ cm and the pipe thickness as $t=1.0$ mm. As a safety factor, the working yield stress is usually taken as about one half of the actual one. The limiting pressures corresponding to the working yield stress have also been included in the figure.

The maximum equatorial Lorentz pressure P_m can be estimated using a model to compute or performing an experiment to measure the maximum of $|B\dot{B}|$ during a quench. For the SSC, with a pipe radius of $b=1.66$ cm and copper thickness of 2 mils, the estimated P_m is about 90 psi. Here for the Tevatron with a 1 mil copper coating, the product $t_c b$ is roughly the same. The dipole flux density is 6.6 T, the same as the SSC dipole. Thus, we expect P_m to be about 90 psi also. In operation, the helium is flowing at a pressure of 3 atm, and we certainly expect it to be higher during a quench. From Fig. 3, it is clear that the Tevatron beam pipe with a 1 mil copper coating will definitely crack during a quench. If there is no copper layer, the situation is completely different. The eddy current flowing along the stainless steel beam pipe will be much smaller due to its high resistivity. According to Eq. (4.2), the maximum Lorentz pressure at the equator will be reduced by a factor of $(\sigma_s t+\sigma_c t_c)/\sigma_s t=30.2$ times to $P_m\sim 3.0$ psi. The beam pipe will be safe even when the helium pressure rises to more than 10 atm.

A way to avoid stress failure is to reduce the thickness of the copper coating. A thickness of 0.1 mil or 2.5 μm reduces the maximum Lorentz pressure to $P_m=11.7$ psi. According to the curve corresponding to the working yield stress, Fig. 3 shows that the beam pipe is still safe up to an external helium pressure of ~ 7 atm. However, such a thickness of copper will only increase the growth time for transverse coupled bunch instability by 3.9 times.

We see from Fig. 3 that most of the time failure will occur at the equatorial points corresponding to the first criterion of Eq. (4.3), which can be rewritten as

$$P_m=\frac{2}{3m^2}\left(1-\frac{p}{p_c}\right)(\sigma_{yp}-mp)\ , \tag{4.5}$$

where $m=\bar{b}/t$ is the beam pipe radius to thickness ratio. Together with the definition of the critical buckling pressure p_c in Eq. (4.1), it is easy to observe that the limiting pressure curve is every sensitive to m. If, for example, the pipe radius is increased by a factor of two or the pipe thickness is decreased by a factor of two so that m is doubled, the working limiting curve will starts off from a new critical pressure of ~ 1.63 atm, a factor of 8 smaller and ends at Lorentz pressure $P_m\sim 8$ psi, a factor of 4 smaller.

The Tevatron beam pipe is actually slightly flattened into a four-sided cross section to allow for better helium flow between the beam pipe and the coils. It is not clear whether such flattening will alter the above estimation considerably.

V. BEAM-PIPE TORQUE LIMITATION

In the above section, it was shown that if a copper coating is deposited on the Tevatron beam pipe, it has to be extremely thin and may be of the order $\sim 2.5\ \mu$m, so that the beam pipe will not be cracked by the external helium pressure during a quench. Such a thin coating is undoubtedly difficult and there will be big unevenness. If, for example, the copper thickness varies with the polar angle φ as

$$t(\varphi) = t_c(1 + \epsilon \sin 2\varphi) , \tag{5.1}$$

the torque per unit length per radius squared twisting the beam pipe about its axis during a quench is given by[5]

$$\frac{\tau}{\ell \bar{b}^2} = \frac{1}{2}\pi\epsilon P_m . \tag{5.2}$$

This torque is resisted by the keys attached to the beam pipe. If the unevenness is big enough, these keys can suffer a stress failure. The detail depends on the actual design and material of the keys. This picture is illustrated in Fig. 4.

If a tensile stress failure occurs, the limiting torque per unit length and radius squared is given roughly by[4]

$$\frac{\tau}{\ell \bar{b}^2} = \frac{w^2 g}{3\ell b h}\sigma_{yp} , \tag{5.3}$$

where each key is assumed to have a width w, length g, height h and located at interval ℓ along the length of the beam pipe. Taking a key similar to that in the SSC, $w \sim 0.2$ in., $g \sim 3$ in., $h \sim 3$ mm, $\ell \sim 18$ in., and $\sigma_{yp} \sim 5000$ psi for the key material, we obtain $\tau/\ell\bar{b}^2 \sim 69$ psi. If as a safety factor, the working yield stress is again taken as one half the actual one, then with the equatorial Lorentz pressure of $P_m \sim 90$ psi per mil of copper layer thickness as estimated in Section VI, Eqs. (5.2) and (5.3) lead to

$$t_c\epsilon \approx 0.24 \text{ mil} . \tag{5.4}$$

Since the copper thickness is of the order 0.1 mil, it takes an unevenness of 240% to break the position-fixing keys. Such unevenness is very unlikely and so is the stress failure of the keys. Such a result in fact is expected. To attain a torque strong enough to break the keys, a certain copper thickness at polar angles $+\pi/4$ and $-5\pi/4$ must exceed that at $-\pi/4$ and $+5\pi/4$ by $t_c\epsilon$. Thus, the unevenness has to be large for a thin coating and vice versa.

VI. CONCLUSIONS

From the above analysis, we conclude that:

(1) If the number of particles per bunch is $N = 2.0 \times 10^{10}$ and betatron tune $\nu_\beta = 20.6$, the worst growth time for transverse coupled-bunch instability is $\tau = 12.9$ ms, which can be controlled by a transverse damper. The parasitic heating due to wall resistivity is $P = 2.2$ kW, which amounts to only 10% of the load of the refrigerators. Therefore, no internal copper coating of the beam pipe is necessary. However, if the number of particles per bunch is increased to $N = 1.0 \times 10^{11}$, the growth time will be reduced to $\tau = 2.6$ ms and the power loss will increase to $P = 56.2$ kW. Both of these numbers then become intolerable. If the betatron tune is 20.9 instead, the worst growth time will be reduced to 3.2 ms and become intolerable.

With a copper layer, the coupled-bunch growth times will be increased by a factor of $\sim \sigma_c t_c / \sigma_s t$ or 29.2 per mil of coating. The parasitic heating of the beam pipe will be reduced by a factor of $\sqrt{\sigma_c/\sigma_s} \sim 34$, roughly independent of the thickness of the coating unless the coating thickness is thinner than ~ 10 μm (0.4 mil).

(2) Because of the large beam-pipe-radius-to-thickness ratio, in order that stress failure of the beam pipe will not occur with the combined action of the exterior helium pressure and the horizontal Lorentz pressure arising from eddy currents in the copper layer during a quench, the thickness of the copper layer has to be $\lesssim 2.5$ μm. With such a thin layer, there should not be any problem about breaking the position-fixing keys due to the unevenness of the coating. This thin coating will still reduce the parasitic power loss by 12.3 times. However, it will not be very effective in controlling transverse coupled-bunch instability. It raises the growth time by a factor of 3.9 only.

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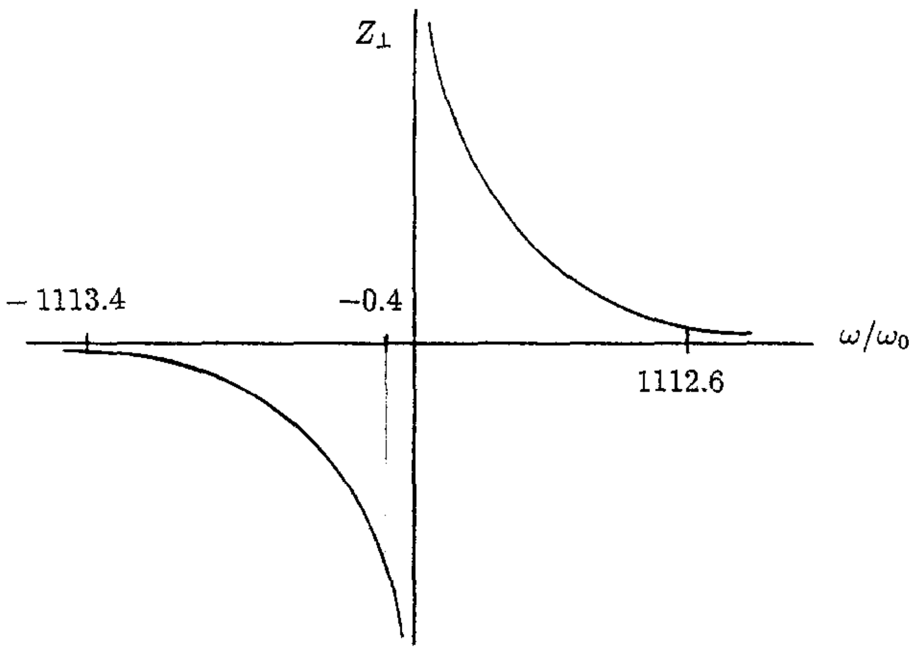

Fig. 1. Plot of the transverse wall impedance $Z_\perp$ and the spectral lines of collective transverse coupled-bunch motion of mode $\mu = 21$. We assume that all the 1113 rf buckets are filled with bunches of equal population.

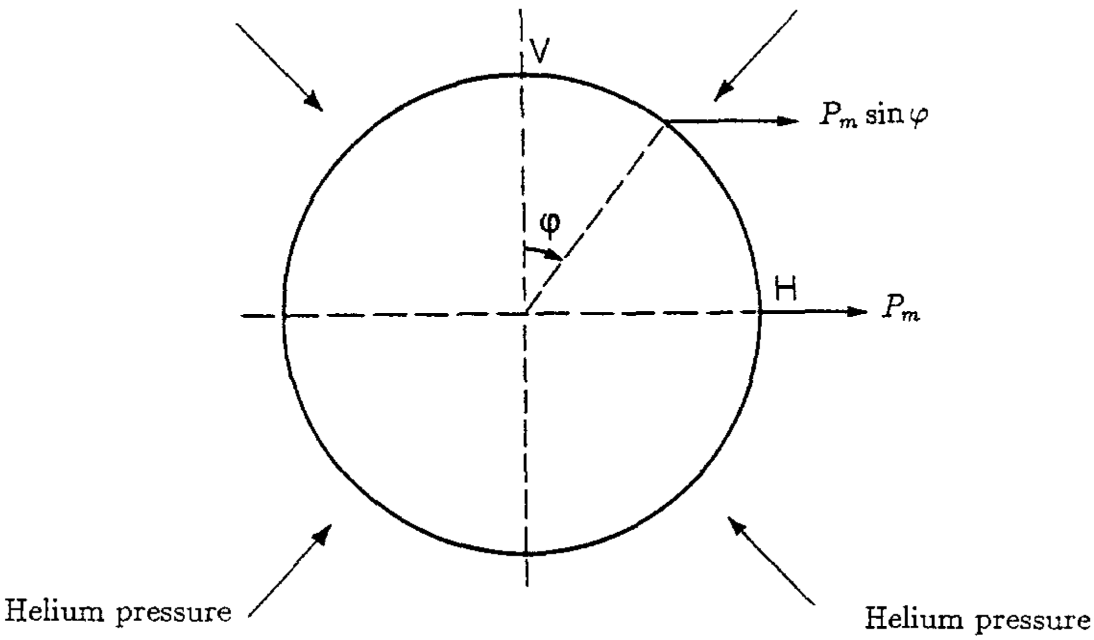

Fig. 2. The pressure produced by the horizontal Lorentz forces acting on the eddy currents in the thin copper layers varies sinusoidally with the polar angle φ, the angle with respect to the direction of the dipole magnetic flux density. Also shown is the radially inward exterior helium pressure.

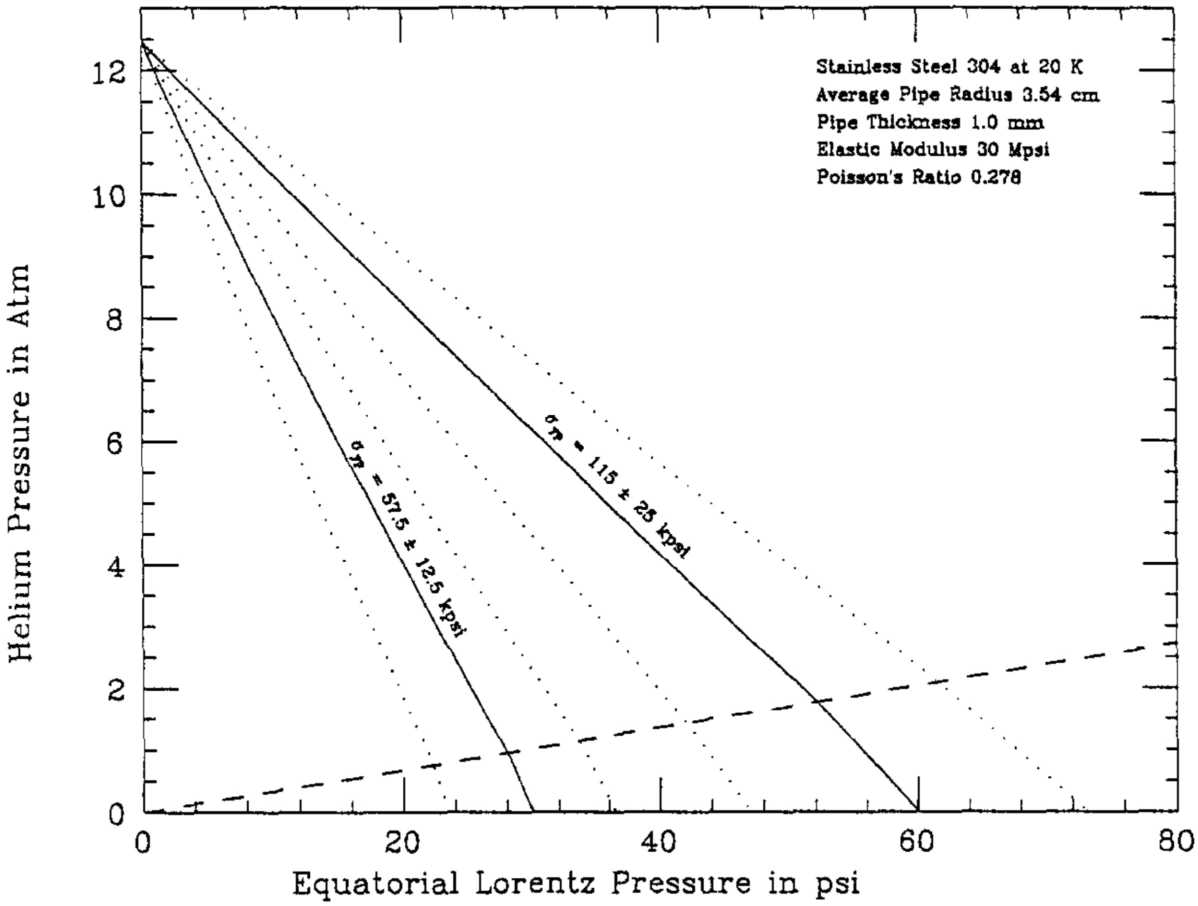

Fig. 3. The limiting helium and Lorentz pressures corresponding to the expected stainless steel yield-point stress of $\sigma_{yp} = 115$ kpsi (upper curves). The dotted curves correspond to the uncertainty of ± 25 kpsi in σ_{yp}. The lower curves result if a safety factor of two in σ_{yp} is applied.

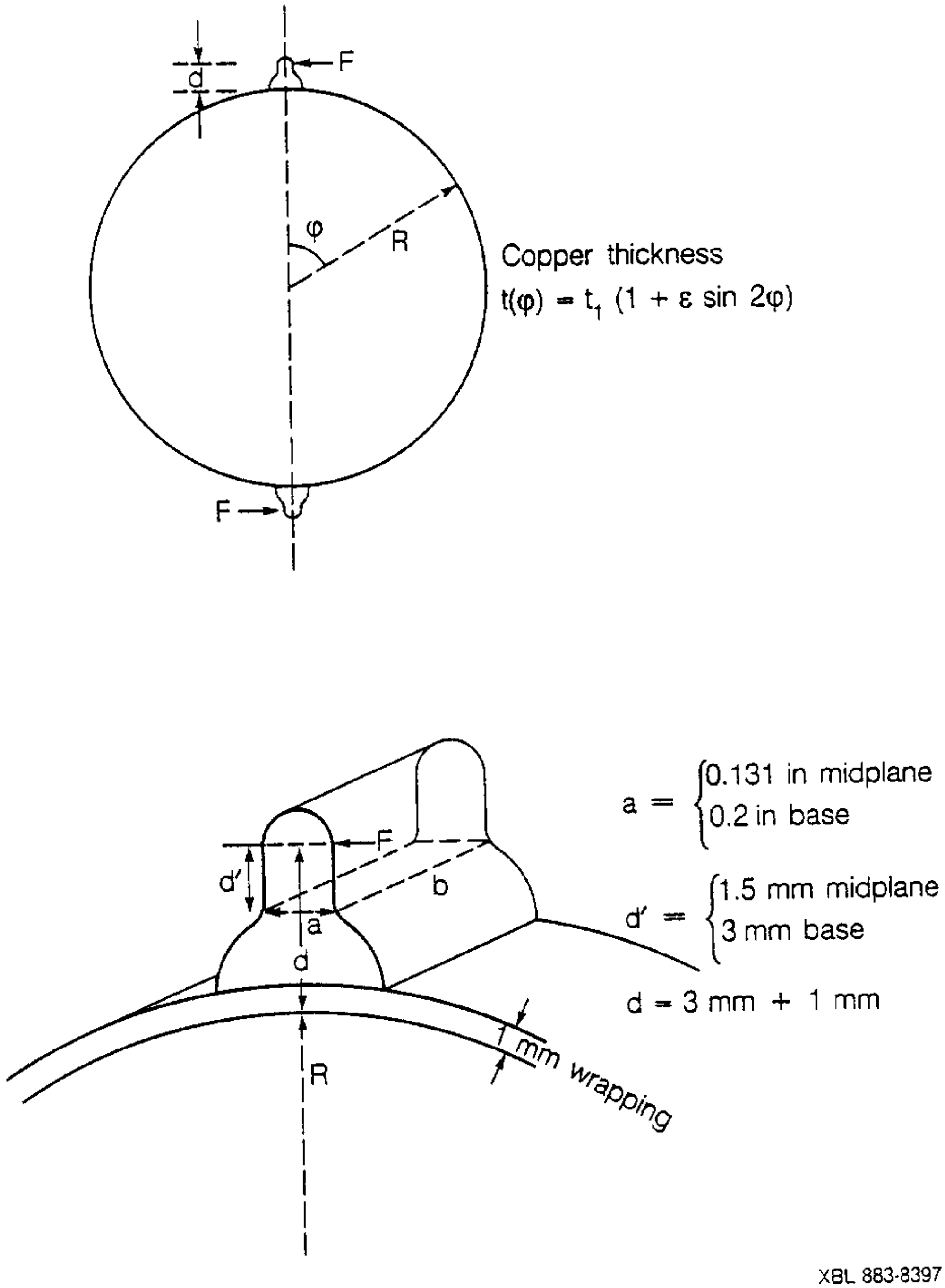

Fig. 4. Geometry of the keys that fix the angular position of the beam pipe with respect to the magnet collars and resist the Lorentz torque that could be produced by non-uniform copper thickness.